

## Polarisation rotation in resonant emission of semiconductor microcavities

A. Kavokin<sup>\*1</sup>, G. Malpuech<sup>2</sup>, P. G. Lagoudakis<sup>2</sup>, J. J. Baumberg<sup>2</sup>, and K. Kavokin<sup>3</sup>

<sup>1</sup> LASMEA, UMR-6602, Université Blaise Pascal, 24 av. des Landais, 63177 Aubiere, France

<sup>2</sup> Department of Physics and Astronomy, University of Southampton, Southampton SO17 1BJ, UK

<sup>3</sup> School of Physics, University of Exeter, Stocker Road, Exeter, EX4 4QL, UK

Received 25 May 2002, accepted 22 June 2002

Published online 18 February 2003

PACS 71.35.Gg, 71.36.+c

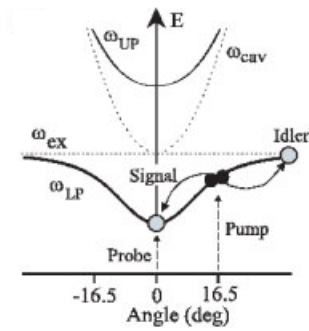
We present the semi-classical theory of the non-linear propagation of polarized light in semiconductor microcavities. This formalism explains the mysterious rotation of the polarization plane of light emitted by a microcavity in the regime of stimulated scattering observed recently. The model describes the stimulated four-wave mixing in microcavities. We show that the exciton spin-splitting induced by the polarized pumping is responsible for giant resonant Faraday rotation of the polarization plane of light emitted by the cavity.

Semiconductor microcavities have attracted a huge new wave of interest since the discovery of the stimulated scattering of exciton-polaritons [1]. This effect has revealed the bosonic nature of the exciton-polaritons, i.e. the half-matter, half-light quasiparticles formed due to the coupling of an exciton resonance with a light mode. A possibility to Bose-condense the polaritons in microcavities that would open a way towards a new generation of opto-electronic devices (e.g. polariton lasers) is widely discussed now. The polariton spin-dynamics in course of the stimulated scattering has a special importance for understanding of the fundamental principles of this phenomenon. Till now, very little has been known about the spin selection rules of the polariton scattering and previous experiments have shown a number of surprising effects that remain unexplained. This paper reports a complete set of the experimental data on the polarization properties of resonantly scattered exciton-polaritons in a microcavity quantitatively described within an original theoretical model.

Let us start with recalling the main features of the experiment [1], which was the first observation of the stimulated polariton scattering in microcavities. A circularly polarized ( $\sigma^+$ ) pump excited the cavity at the so-called magic angle and generated a coherent polariton population at the inflection point of the lower polariton branch (see Fig. 1). Then a circularly polarized probe generated polaritons in the ground state that stimulated the resonant scattering of polaritons created by the pump pulse to the probed state ( $k=0$ ). At first glance, it seemed that the scattering of polaritons from the pumped state toward the ground state could only happen if the pump and the probe were co-circularly polarized (both  $\sigma^+$  for example). In case of cross-circularly polarized light no stimulation should happen. It seemed possible to describe all the intermediate situations where both pump and probe are elliptically polarized simply de-

---

\* Corresponding author: e-mail: kavokin@lasmea.univ-bpclermont.fr

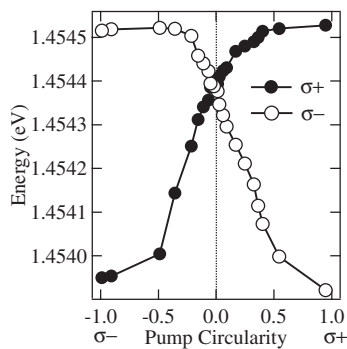


**Fig. 1** Scheme of the resonant pump-probe experiment. Polariton–polariton stimulated scattering schematically depicted on the dispersion curve of the lower branch of the exciton-polaritons in the microcavity under study.

composing the pulse pump on the  $\sigma^+$  and  $\sigma^-$  components one of which is effectively scattered and another one is not scattered. This simple picture has however been completely ruled out by recent experimental results of P. Lagoudakis et al. [2] who have reported extremely unusual polarization properties of a microcavity excited resonantly at the “magic angle”. To summarize briefly their results, in case of the linear polarized pump pulse and circularly polarized probe pulse they have observed the linearly polarized signal having a polarization plane rotated by 45 degrees with respect to the pump polarization. In case of elliptically polarized pump pulse, the signal becomes as well elliptical while the direction of the main axis of the ellipse rotates as a function of the circularity of pump. In the case of purely circular pump, the polarization of the signal is also circular but it's intensity is twice less than in the case of a linear pump. The polarization of the idler signal scattered at the twice-magic angle shows the similar behavior, while in the case of a linearly polarized pump the idler polarization is rotated by 90 degrees with respect to the pump polarization.

In the paper by Lagoudakis a qualitative interpretation of a part of these data is done in terms of the optical Josephson effect and stimulated spin-flip processes in the cavities. However, a detailed understanding of the observed effects has not been achieved.

In this paper we present the semi-classical theory of the non linear propagation of polarized light in semiconductor microcavities together with a new large set of the experimental data on the resonant spin polarization effects in microcavities. Our model consists of two parts. We consider first the diffraction of the pump-pulse on the optical grating created by the pump and probe pulses in a framework of a classical non-linear optics formalism. The self-diffraction of the pump is found to be responsible for the rotation by 45 degrees of the polarization when the pump is linearly polarized. Then the diffracted pulse then propagates freely within the cavity before being reemitted outside. This propagation is described using the generalized scattering state technique [4, 5]. The non-linear aspect of the light propagation in a strongly pumped cavity is accounted through the renormalization of the energies of the exciton resonances due to the band filling effect [6]. A large ( $\sigma^+$ ,  $\sigma^-$ ) coherent polariton population is generated by the pump laser. This large coherent population provokes a blue shift of the ( $\sigma^+$ ,  $\sigma^-$ ) exciton resonances because of their self-repulsive interaction. This is evidenced on Fig. 2, which shows the shift of the  $\sigma^+$



**Fig. 2** Energies of  $\sigma^-$  and  $\sigma^+$  exciton resonances as functions of the circularity of pumping.

and  $\sigma^-$  polariton lines as a function of the pump circularity. The dependence of ( $\sigma^+$ ,  $\sigma^-$ ) energies is smooth when the corresponding pump density is small. It then becomes sharper before saturating when the pump circularity is close to one. These data are in a good agreement with the results of Ref. [7] for a CdTe-based microcavity. As we will show below, this shift is responsible of a giant Faraday rotation of the polarization plane of light during its propagation within the cavity. This Faraday rotation coupled to the initial rotation produced by the diffraction of the probe is mainly responsible of the experimental finding of [2]. We will assume that on the time scale of the experiment (units of picoseconds) the polariton spin is conserved.

We represent an electric field of any wave propagating in our structure as a vector:

$$\mathbf{E} = \begin{bmatrix} E_x \\ E_y \end{bmatrix}, \quad (1)$$

having the in-plane components  $E_x, E_y$ .

The electric field of the four-wave mixing signal is given by

$$E_\alpha^{\text{sig}} = T_{\alpha\beta\gamma\delta} P_\beta P_\gamma S_\delta, \quad (2)$$

where  $\alpha, \beta, \gamma, \delta$  take the values  $x$  or  $y$ ,  $P_\alpha$  are the components of the pump pulse,  $S_\alpha$  are the components of the probe pulse.

In order to obtain the tensor  $T$  let us analyze the two-component matrix

$$M_{\alpha\delta} = T_{\alpha\beta\gamma\delta} P_\beta P_\gamma. \quad (3)$$

For the symmetry reasons, it can be represented in a way:

$$M_{\alpha\beta} = A(P_x P_x^* + P_y P_y^*) \delta_{\alpha\beta} + B(P_\alpha P_\beta^* + P_\beta P_\alpha^*) + C(P_\alpha P_\beta^* - P_\beta P_\alpha^*), \quad (4)$$

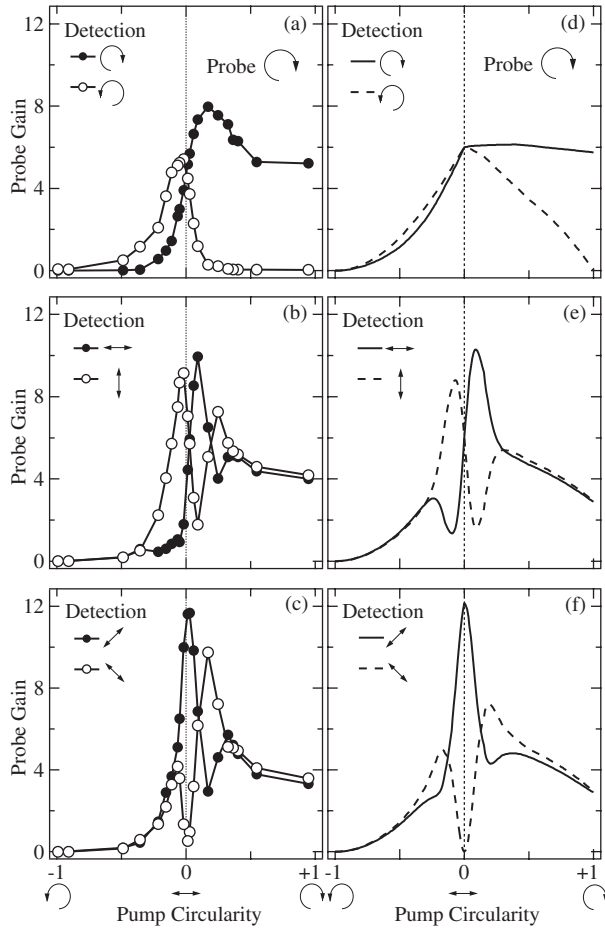
where  $A, B, C$  are constants,  $*$  denotes the complex conjugation,  $\delta_{\alpha\beta} = 1$  if  $\alpha = \beta$ , otherwise it is zero. The first term in the right part of Eq. (4) describes the isotropic optical response of the systems, the second term yields the in-plane anisotropy induced by the pump pulse, and the third term describes the pump-induced gyrotropy.

The gyrotropy comes from the spin-splitting of the exciton resonance in case of the circular or elliptical pumping. As mentioned in the introduction, the splitting of the exciton resonance in  $\sigma^+$  and  $\sigma^-$  polarizations does not only play a role during the non-linear parametric scattering, but it also influences the linear optical response of the quantum well. We'll take this effect into account while calculating the linear propagation of light in the cavity. Putting  $A = 1$  we reduce the number of the unknown parameters of the problem to two. The arguments justifying our choice of parameters  $B$  and  $C$  will be given in the last part of the paper. Now we just give the values that are:

$$B = (i-1)/2, \quad C = i/8 \text{ for the signal}, \quad B = -1/2, \quad C = 2i/3 \text{ for the idler}.$$

The stimulation itself is accounted phenomenologically. We simply assume that the scattering process has an efficiency equal to zero for those exciton-polariton which have spins antiparallel to those of the excitons created by the probe pulse. In the case of  $\sigma^+$  polarized probe pulse (that is the experimental case) we multiply thus the matrix (4) by a function:

$$f = 1 - I^- / (I^- + I^+) \Theta(I^- - I^+), \quad (5)$$



**Fig. 3** Emitted signal intensities decomposed into (a, d) circular, (b, e) linear and (c, f) linear diagonal polarizations, as a function of the circularity of pump: experiment (a, b, c) and theory (d, e, f). The probe is  $\sigma^+$  polarized.

where  $I^-$ ,  $I^+$  are the intensities of  $\sigma^-$  and  $\sigma^+$  components of the pump-pulse, respectively,  $\Theta(x) = 1$ , if  $x > 0$ , and 0 otherwise.

Figure 3 show the intensities of the signal, versus the circularity of the pump pulse measured experimentally in different polarizations (a, b, c), and calculated within the model described above (d, e, f). The circularity is defined as  $\rho = \frac{I^+ - I^-}{I^+ + I^-}$ . One can see the excellent agreement between the theory and the

experiment. The signal polarization rotated by  $45^\circ$  with respect to the polarization plane of the pump pulse if the latter is linearly polarized is well described by the matrix (4). The fast rotation of the polarization is also correctly described. It can thus be definitively attributed to the giant Faraday rotation of the light polarization during its propagation within the cavity, Faraday rotation caused by the by the pump-induced spin splitting of the exciton resonance (see Fig. 1).

This conclusion is supported by the following analysis that described analytically the rotation of the light polarization in the cavity versus the spin splitting. In this framework, the transmission coefficient of the quantum well in the vicinity of a (split) exciton resonance is given by

$$t_{\sigma^+, \sigma^-} = 1 + \frac{i\Gamma_0}{\omega_0^{\sigma^+, \sigma^-} - \omega - i(\gamma + \Gamma_0)}, \quad (6)$$

where  $\omega_0^{\sigma^+, \sigma^-}$  is the exciton resonance frequency in two polarizations,  $\Gamma_0$  is the exciton radiative decay rate,  $\gamma$  is the exciton non-radiative decay rate. If  $\delta \ll \gamma + \Gamma_0$  the polarization plane of a linearly polarized light passing through the QW rotates by the angle [8]

$$\varphi = \frac{(\omega_0^{\sigma^-} - \omega_0^{\sigma^+}) \Gamma_0}{(\gamma + \Gamma_0)^2} . \tag{7}$$

In a microcavity the light emitted by a QW circulates many times between the mirrors thus accumulating the rotation before escaping the cavity.

The amplitude of the emitted light can be found as

$$E = t_1 + t_1 r_1 e^{i\varphi} + t_1 r_1^2 e^{2i\varphi} + \dots = \frac{t_1}{1 - r_1 e^{i\varphi}} , \tag{8}$$

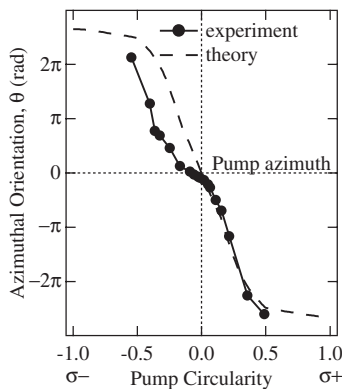
where  $r_1$  and  $t_1$  are the reflection and transmission coefficient of the Bragg mirror, respectively. The angle of resulting rotation of the linear polarization is

$$\theta = \arg(E) = \frac{r_1 \sin \varphi}{1 - r_1 \cos \varphi} , \tag{9}$$

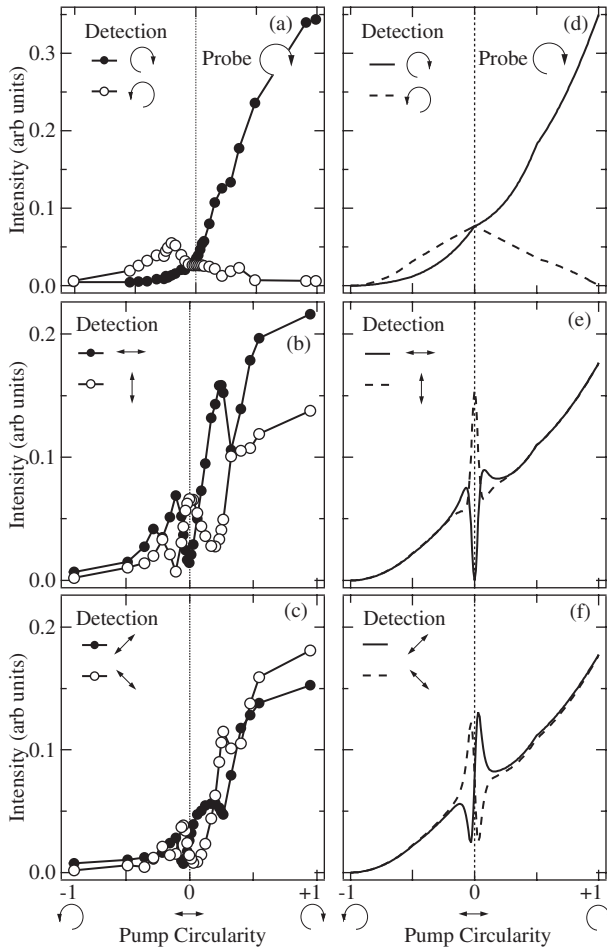
Figure 4 shows the evolution of  $\theta$  vs. the pump circularity. It clearly confirms that the fast rotation of the polarization observed experimentally is due to the Faraday rotation of the light polarization, itself induced by the split polariton resonance.

Figure 5 shows the same as Fig. 1, but for the idler. The polarization is now found to rotate by  $90^\circ$  when the pump is linear. The oscillations remain, even if they are partially masked by the background emission. Another remarkable fact is that the intensity of the signal is now higher when both pump and probe are circular, by contrast with what happens for the signal. It should be pointed out at this stage, that the idler polariton life-time is much longer than the signal life time. Spin relaxation and dephasing processes certainly play a non-negligible role in this case. Despite of this more complex situation, the theory-experiment agreement stays rather good. In particular, the overall intensity evolution versus the pump circularity and the  $90^\circ$  rotation of the linear pump are correctly described.

In this last part, we discuss the choice (5, 6) of the parameters  $B$  and  $C$  for the signal and idler. The signal and idler emission are due to the polariton–polariton scattering process. As has been shown by Ciuti et al. [3], the matrix element of this interaction is dominated by the term coming from the exciton-exciton exchange interaction. Thus, the dipole moments of two polaritons that appear after the scattering act should be oriented along the axis connecting two initial polaritons. In the case of linear pumping, the initial dipole moments of the interacting excitons are aligned along the  $X$ -direction. Since the polaritons move in the  $XY$ -plane and since the dipole-dipole interaction is strongly anisotropic, the polarization of



**Fig. 4** Rotation angle of the linear polarisation of signal as a function of the circularity of pump.

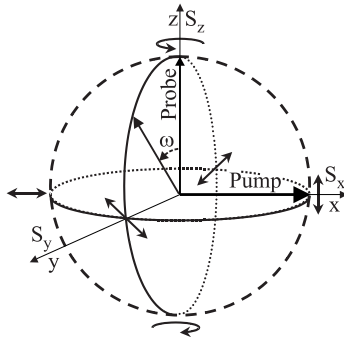


**Fig. 5** Emitted idler intensities decomposed into (a, d) circular, (b, e) linear and (c, f) linear diagonal polarisations, as a function of the circularity of pump: experiment (a, b, c) and theory (d, e, f). The probe is  $\sigma^+$  polarized.

two resulting polaritons will be preferentially in  $Y$ -direction. This is what the idler emission shows. The polarization of a signal pulse is also dependent on the polarization of the probe pulse. Actually, the stimulated scattering assumes that the polaritons generated by the pump pulse scatter to the same quantum state as that of the probe pulse. However, in the case of linearly polarized pump and circularly polarized probe, this process is formally forbidden as non-conserving the spin. On the other hand, due to the exchange interaction between polaritons from pump and probe the polaritons excited by the probe pulse might start rotating their spins (see the scheme in Fig. 6). This process is described by the pseudospin Hamiltonian [9]

$$\hat{H} = \frac{\hbar}{2} (\omega_x \sigma_x + \omega_z \sigma_z), \tag{10}$$

where  $\sigma_i$  are the Pauli matrices,  $\hbar\omega_x$  is the pump-induced splitting of the exciton states linearly polarized along  $x$  and  $y$  directions,  $\hbar\omega_z$  is the pump induced splitting of the exciton states circularly polarized  $\sigma^+$  and  $\sigma^-$ . The second term in the right part of Eq. (10) is zero in the case of linearly polarized pumping, while the first term is non-zero and provides the rotation of the pseudospin of the probe polaritons in the  $(z, y)$  plane. That is why the probe pseudospin is starting to have the  $y$ -component that corresponds to



**Fig. 6** Rotation of the pseudospin of the probe pulse around the effective magnetic field created by the pump pulse shown schematically on the Poincaré sphere.

the diagonal polarization in-plane [8]. Expressing the projections of the pseudospin of the probe-pulse on  $x$ - and  $y$ -axes with use of the Hamiltonian (10), one can express the signal amplitude as:

$$\begin{bmatrix} E_x^{\text{sig}} \\ E_y^{\text{sig}} \end{bmatrix} = \sqrt{\frac{\rho}{2}} \begin{bmatrix} 1 \\ i \end{bmatrix} + \xi \chi_x \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \xi \chi_y \begin{bmatrix} 1 \\ \pm 1 \end{bmatrix}. \tag{11}$$

Here  $\tau$  is the rotation time of the probe-pulse,  $\chi_{x,y}$  are the projections of the rotating pseudospin of the polaritons created by the probe pulse on  $x$  and  $y$  axes given by

$$\chi_x = \rho \sqrt{1 - \rho^2} (1 - \cos \omega \tau),$$

$$\chi_y = \sqrt{1 - \rho^2} \sin \omega \tau,$$

where

$$\omega = \sqrt{\omega_x^2 + \omega_z^2}.$$

The normalization constant

$$\xi = [(1 + \rho) (\rho^2 (1 - \cos \omega \tau)^2 + 2 \sin^2 \omega \tau)]^{-1/2}.$$

Plus or minus sign in the right-hand side of Eq. (11) correspond to the positive or negative value of the coefficient  $\chi_y$ . Here we assumed that due to the spin conservation requirement, the percentage of the polaritons that contribute to the circularly polarized component of signal is  $\rho$ , while  $(1 - \rho)$  polaritons contribute to the linearly polarized signal emission.

Comparing Eqs. (11) and (2–4), one can obtain the coefficients  $A, B, C$ . A remarkable fact is that the coefficients  $A$  and  $B$  are independent on the parameter  $\omega \tau$  and are given by  $A = 1, B = (i - 1)/2$ . The coefficient  $C$  changes as a function of  $\omega \tau$ .

For the idler, Eq. (11) cannot be applied since there is no bosonic stimulation of the scattering to the idler polariton states. The idler polaritons are initially all preferentially  $Y$ -polarized (because of the directional anisotropy of the dipole–dipole interaction, as discussed above). Then, the light emitted in the idler direction experiences the Faraday rotation that leads to the oscillations in the idler polarization as a function of  $\rho$ .

The pseudospin model presented above is confirmed by the following experimental observations:

i) The strongly enhanced signal emission is observed at delay times between the pump and probe pulses exceeding 1 ps approximately in the case of the linear pump and the circular probe. This confirms

our assumption that the probe pulse should have a  $Y$ -component of the pseudospin to stimulate the scattering. This  $Y$ -component is initially zero but appears with a 1 ps delay as a result of the pseudospin precession.

ii) The signal emission is non-polarized in the cw regime, because in this case the rotation of the probe pseudospins during a long time yields equal populations of both diagonal polarizations. Note, that at the same time the idler remains  $y$ -polarized.

In conclusion, we have presented a phenomenological model that allows to explain the surprising behavior of the polarized emission of resonantly excited microcavities. We show that the effect is governed by the diffraction of the pump pulse on the polarization grating created by the pump and probe pulses and by the resonant Faraday rotation of the polarization plane of light propagating in the cavity in the vicinity of the spin-split exciton resonance. The rotation of the polarization plane of the emitted light by  $45^\circ$  or  $90^\circ$  (for signal and idler) with respect to the polarization of the pump pulse at linearly polarized pumping is a consequence of the specific polarization selection rules imposed by the diffraction of the pump pulse, while the oscillations of polarization of signal and idler as a function of pump circularity is a resonant Faraday rotation effect. We present a microscopic model explaining the signal and idler polarization behavior at the linear pump.

**Acknowledgement** We have greatly benefited from the discussions with M.I. Dyakonov, who, in particular, has proposed to us to represent the matrix  $M_{\alpha\beta}$  in form (4). The work has been supported by the EU RTN “CLERMONT” program, contract No. HPRN-CT-1999-00132.

## References

- [1] P. G. Savvidis et al., Phys. Rev. Lett. **84**, 1547, (2000).
- [2] P. Lagoudakis et al., Phys. Rev. B **65**, 161310 (R) (2002).
- [3] C. Ciuti et al., Phys. Rev. B **58**, 7926 (1998).
- [4] G. Malpuech et al., Phys. Rev. Lett. **85**, 650 (2000).
- [5] G. Malpuech and A. V. Kavokin, Semicond., Sci. and Technol. Topical Review, **16**, (3), R1–R23, (2001).
- [6] J. Fernandez-Rossier and C. Tejedor, Phys. Rev. Lett. **78**, 4809 (1997).
- [7] M. D. Martin et al., phys. stat. sol. (a), **190**, 351 (2002).
- [8] A. Kavokin et al., PRB **56**, 1087 (1997).
- [9] R. I. Dzhioev et al., Phys. Rev. B **56**, 13405 (1997).